FLOATING POINT OPERATIONS

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CS/ECE 3810: Computer Organization

Overview

- This lecture
  - Floating point operations
    - Addition
    - Multiplication
  - Floating point instructions

Floating Point Addition

- Numbers maintain only 4 decimal digits and 2 exponent digits
    - $9.999 \times 10^{1} + 1.610 \times 10^{-1}$

Floating Point Addition

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- Convert to the larger exponent
  - $9.999 \times 10^{1} \quad + \quad 0.016 \times 10^{1}$

Floating Point Addition

- Numbers maintain only 4 decimal digits and 2 exponent digits
    - $9.999 \times 10^{1} + 1.610 \times 10^{-1}$

- Convert to the larger exponent
    - $9.999 \times 10^{1} + 0.016 \times 10^{1}$
- Add
    - 10.015×10^{1}

Floating Point Addition

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  - $9.999 \times 10^1 + 1.610 \times 10^{-1}$

  - Convert to the larger exponent
    - $9.999 \times 10^1 + 0.016 \times 10^1$
  - Add
    - 10.015×10^1
  - Normalize
    - 1.0015×10^2
  - Check for overflow/underflow
  - Round
    - 1.002×10^2
  - Re-normalize

Floating Point Addition

- Numbers maintain only 4 decimal digits and 2 exponent digits
    - $9.999 \times 10^1 + 1.610 \times 10^{-1}$

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      - 1.002×10^2
  - Re-normalize

If we had more fraction bits, these errors would be minimized

Floating Point Addition

- Numbers maintain only 4 binary digits and 2 exponent digits
  - $1.010 \times 2^1 + 1.100 \times 2^3$

- Convert to the larger exponent
  - $0.0101 \times 2^3 + 1.100 \times 2^3$
- Add
  - 1.1101×2^3
- Normalize
  - 1.1101×2^3
- Check for overflow/underflow

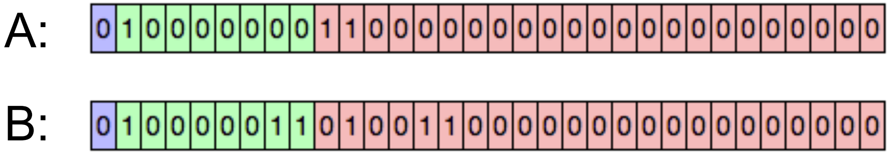
Floating Point Addition

- Numbers maintain only 4 binary digits and 2 exponent digits
    - $1.010 \times 2^1 + 1.100 \times 2^3$
  - Convert to the larger exponent
    - $0.0101 \times 2^3 + 1.100 \times 2^3$
  - Add
    - 1.1101×2^3
  - Normalize
    - 1.1101×2^3
  - Check for overflow/underflow
  - IEEE 754 format (32-bit)

0 10000010 11010000000000000000000

Floating Point Addition

- **Example:** add the following two single-precision floating point numbers.



A: 0 1 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

B: 0 1 0 0 0 0 0 1 1 0 1 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

Steps:

1. Convert to larger exponent
2. Add
3. Normalize
4. Round

Floating Point Addition

- **Example:** add the following two single-precision floating point numbers.

A:	0	1	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B:	0	1	0	0	0	0	0	1	1	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$E_A = 128 \quad M_A = 1.11_{two}$

$E_B = 131 \quad M_B = 1.010011_{two}$

Floating Point Addition

- Example: add the following two single-precision floating point numbers.

A: 01000000011000000000000000000000

B: 01000001101001100000000000000000

$E_A = 128$ $M_A = 1.11_{two}$ $E_B = 131$ $M_B = 1.010011_{two}$

$E_A = 131$ $M_A = 0.00111_{two}$ $E_B = 131$ $M_B = 1.010011_{two}$

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- **Example:** add the following two single-precision floating point numbers.

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B:	0	1	0	0	0	0	0	1	1	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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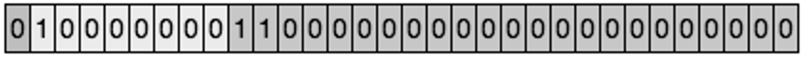
$E_A = E_B = 131$ $\qquad$ $M_A + M_B = 0.00111_{two} + 1.010011_{two} = 1.100001_{two}$

Floating Point Addition

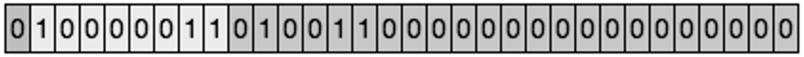
- Example: add the following two single-precision floating point numbers.

A:



0 1 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

B:



0 1 0 0 0 0 0 1 1 0 1 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

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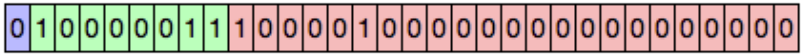
A + B:



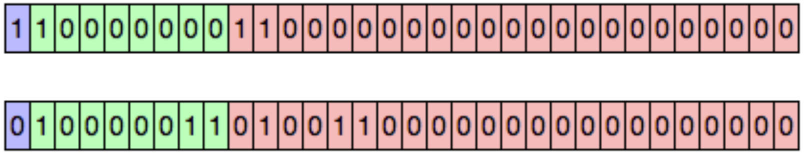
0 1 0 0 0 0 0 1 1 1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

Floating Point Multiplication

- Similar steps are required for multiplication
  - Compute exponent
    - Need to remove bias
  - Multiply significands
    - May end up unnormalized
  - Normalize
    - Shift the point
  - Round
    - Fit in the number of bits
  - Assign sign
    - Compute sign

Floating Point Multiplication

- **Example:** multiply the following two single-precision floating point numbers.



A: 1 1 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

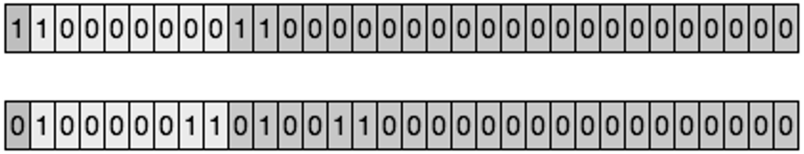
B: 0 1 0 0 0 0 0 1 1 0 1 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

Steps:

1. Compute exponent
2. Multiply significands
3. Normalize
4. Round
5. Compute sign

Floating Point Multiplication

- **Example:** multiply the following two single-precision floating point numbers.



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- **Example:** multiply the following two single-precision floating point numbers.

A: 11000000011000000000000000000000

B: 01000001101001100000000000000000

$E_A = 128$ $M_A = 1.11_{two}$ $E_B = 131$ $M_B = 1.010011_{two}$

$E_{AxB} = 128 + 131 - \mathbf{127} = 132$ $M_{AxB} = 10.01000101_{two}$

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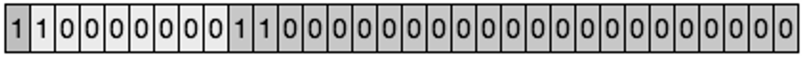
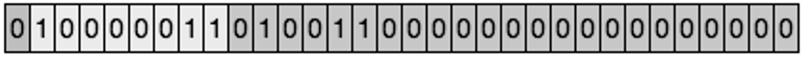
A: 
1 1 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

B: 
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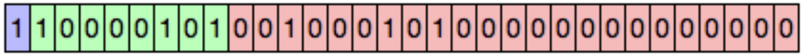
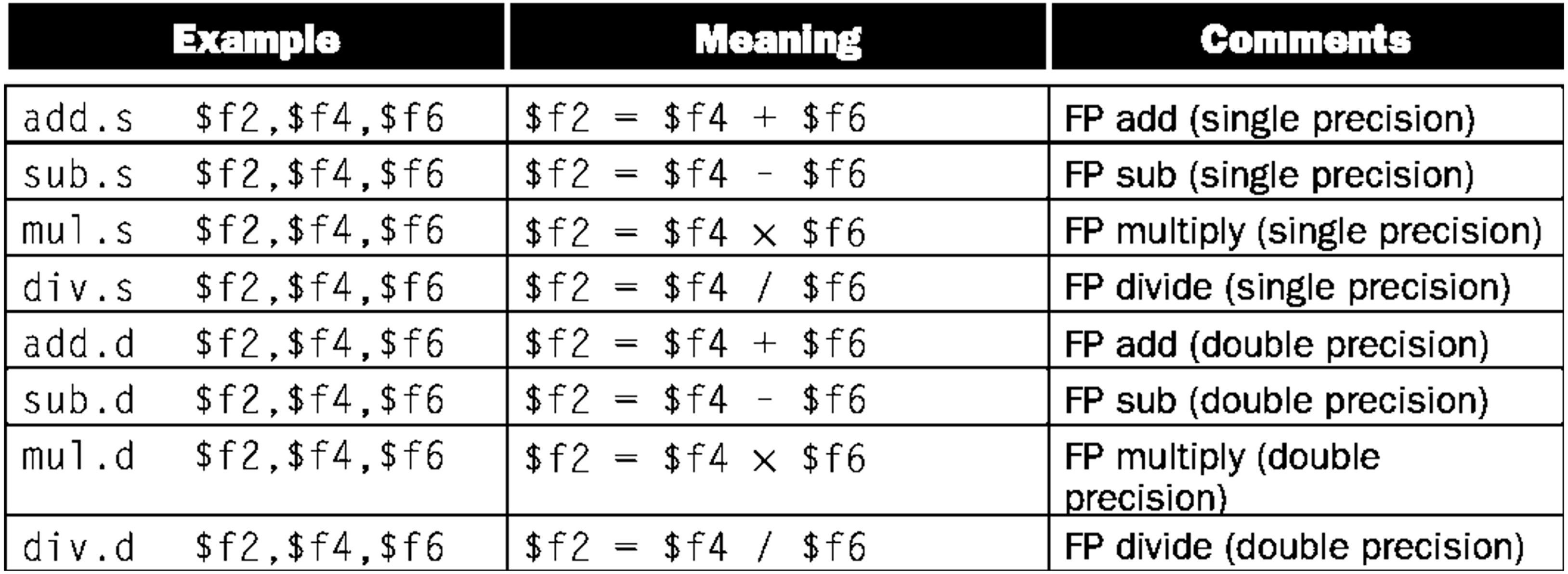
A x B: 
1 1 0 0 0 0 1 0 1 0 0 1 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0

Floating Point Instructions

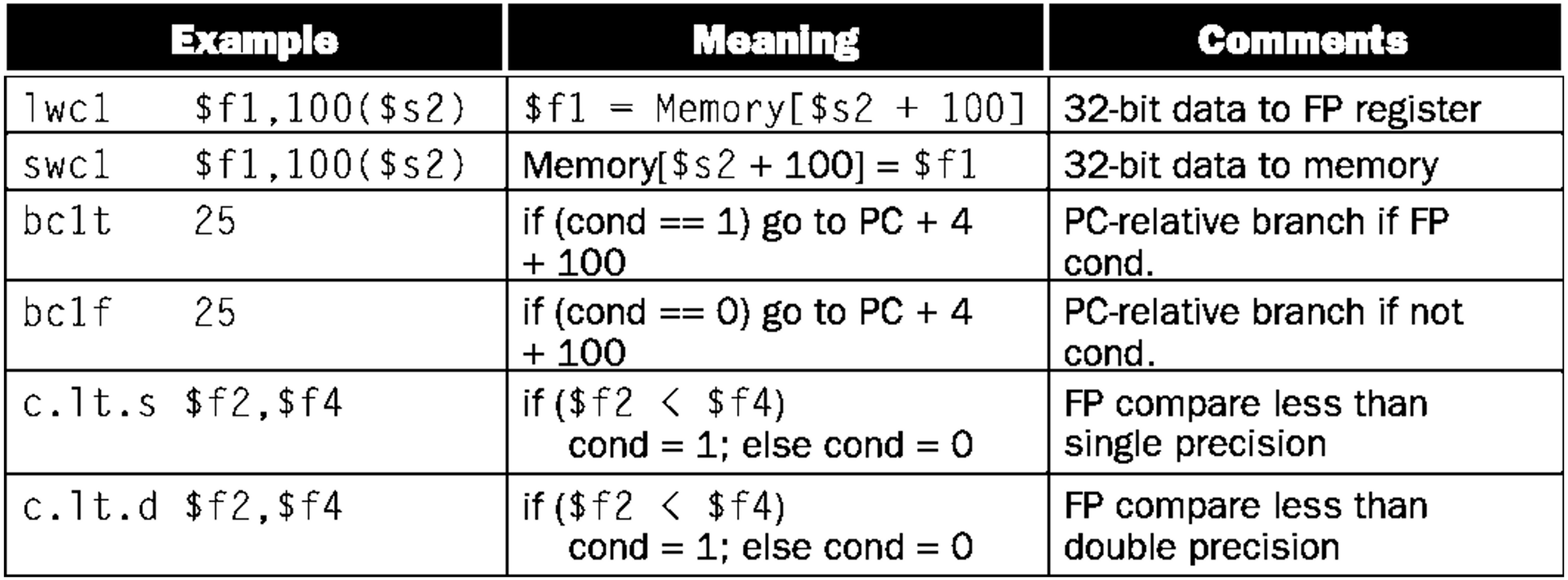
- MIPS employs separate registers for floating point
  - 32-bit registers: $f0, $f1, …, $f31.
  - Each register represents a single-precision number
  - Register pairs are used for double-precision
    - Example: $f0 refers to {$f0, $f1}



Example	Meaning	Comments
`add.s $f2,$f4,$f6`	`$f2 = $f4 + $f6`	FP add (single precision)
`sub.s $f2,$f4,$f6`	`$f2 = $f4 - $f6`	FP sub (single precision)
`mul.s $f2,$f4,$f6`	`$f2 = $f4 × $f6`	FP multiply (single precision)
`div.s $f2,$f4,$f6`	`$f2 = $f4 / $f6`	FP divide (single precision)
`add.d $f2,$f4,$f6`	`$f2 = $f4 + $f6`	FP add (double precision)
`sub.d $f2,$f4,$f6`	`$f2 = $f4 - $f6`	FP sub (double precision)
`mul.d $f2,$f4,$f6`	`$f2 = $f4 × $f6`	FP multiply (double precision)
`div.d $f2,$f4,$f6`	`$f2 = $f4 / $f6`	FP divide (double precision)

Floating Point Instructions

- Load/Store instructions by coprocessor 1 (c1)
  - Still use integer registers for address computation
- Comparison instructions
  - Set an internal bit (cond) to be inspected by branch instructions



Example	Meaning	Comments
`lwc1 $f1,100($s2)`	$f1 = Memory[$s2 + 100]	32-bit data to FP register
`swc1 $f1,100($s2)`	Memory[$s2 + 100] = $f1	32-bit data to memory
`bc1t 25`	if (cond == 1) go to PC + 4 + 100	PC-relative branch if FP cond.
`bc1f 25`	if (cond == 0) go to PC + 4 + 100	PC-relative branch if not cond.
`c.lt.s $f2,$f4`	if ($f2 < $f4) cond = 1; else cond = 0	FP compare less than single precision
`c.lt.d $f2,$f4`	if ($f2 < $f4) cond = 1; else cond = 0	FP compare less than double precision

Code Example

- Convert a temperature in Fahrenheit to Celsius

```
float f2c(float fahr) {
  return ((5.0/9.0)*(fahr–32.0));
}
```

  - Assume that constants are stored in global memory

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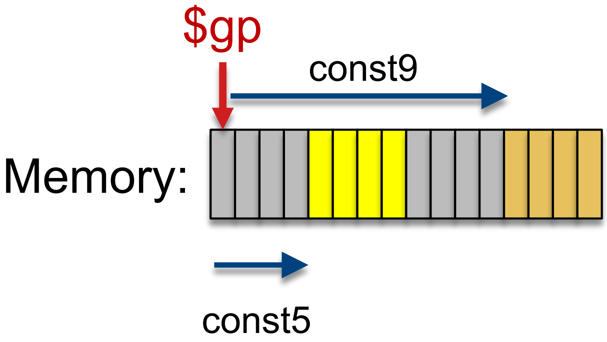

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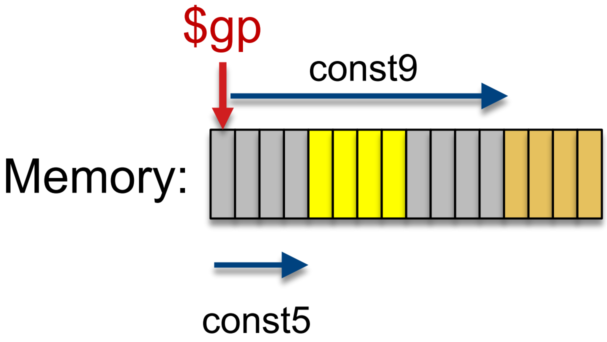

```
f2c: mtc1   $a0, $f12
     lwc1   $f16, const5($gp)
     lwc1   $f18, const9($gp)
     div.s  $f16, $f16, $f18
     lwc1   $f18, const32($gp)
     sub.s  $f18, $f12, $f18
     mul.s  $f0, $f16, $f18
     jr     $ra
```