NUMBER OPERATIONS

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Overview

- This lecture
  - Binary representation
  - Negative numbers
  - Basic operations

Binary Representation

- The binary number

  11011000 00010101 00101110 11100111

  Most significant bit Least significant bit

- The number quantity (decimal)

  $$1 \times 2^{31} + 1 \times 2^{30} + 0 \times 2^{29} + \ldots + 1 \times 2^{0} = 3625266919$$

- A 32-bit word can represent 2^{32} numbers between 0 and 2^{32}-1 (4,294,967,295)
  - Represent only positive numbers
  - Also known as the unsigned representation

Negative Numbers

- The binary number

  11011000 00010101 00101110 11100111

  Sign bit
- Sign-magnitude representation
  - 1. Quantify the magnitude (31 bits)

    $1 \times 2^{30} + 0 \times 2^{29} + \ldots + 1 \times 2^{0} = 1477783271$
  - 2. Determine the sign based in the sign bit

    -1477783271
- Example: 3-bit sing-magnitude
  - How many numbers
  - How to do arithmetic

Negative Numbers

- The binary number

  `11011000 00010101 00101110 11100111`

  Sign bit

- 1's complement: -x is represented by inverting x's bits
  - 1. Invert the bits if the sign bit is set

    `00100111 11101010 11010001 00011000`

  - 2. Quantify the magnitude (31 bits)

    $-1 \times (1 \times 2^{29} + \ldots + 0 \times 2^{0}) = -669700376$

- Example: 3-bit 1's complement
  - How many numbers
  - How to do arithmetic

Negative Numbers

- The binary number

  11011000 00010101 00101110 11100111

  Sign bit

- Sign-magnitude and 1's complement are not favorable
  - Relatively complex implementation of arithmetic operations

- A 32-bit word represents 2^{32}-1 numbers between $-2^{31}+1$ and $+2^{31}-1$
  - Two different representations for zero

Negative Numbers

- The binary number

  11011000 00010101 00101110 11100111

  Sign bit

- 2's complement representation
  - Give the sign bit a negative weight

  $$1 \times -2^{31} + 1 \times 2^{30} + 0 \times 2^{29} + \ldots + 1 \times 2^{0} = -669700377$$

- Example: 3-bit 2's complement
  - How many numbers
  - How to do arithmetic

Negative Numbers

- The binary number

  11011000 00010101 00101110 11100111

  Sign bit

- 2's complement representation
  - Give the sign bit a negative weight

$$1 \times -2^{31} + 1 \times 2^{30} + 0 \times 2^{29} + \ldots + 1 \times 2^{0} = -669700377$$

- A 32-bit word represents 2^{32} numbers between -2^{31} and $+2^{31}-1$.
  - No repeated numbers and simple arithmetic implementation

Example: 2's Complement

- Compute the 32-bit 2's complement representations for the following decimal numbers:
  - 5, -5, -6

Example: 2's Complement

- Compute the 32-bit 2's complement representations for the following decimal numbers:
  - 5, -5, -6
- Given -5, verify that negating and adding 1 yields the number 5
-

5: 0000 0000 0000 0000 0000 0000 0000 0101
-5: 1111 1111 1111 1111 1111 1111 1111 1011
-6: 1111 1111 1111 1111 1111 1111 1111 1010

Example

- All 32-bit 2's complement representations

```
int num = 0;
do {
  num++;
} while(num != 0);
```

$0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000_{two} = 0_{ten}$
$0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0001_{two} = 1_{ten}$
…
$0111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111_{two} = 2^{31}-1$

$1000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000_{two} = -2^{31}$
$1000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0001_{two} = -(2^{31} - 1)$
$1000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0010_{two} = -(2^{31} - 2)$
…
$1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1110_{two} = -2$
$1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111_{two} = -1$

Signed and Unsigned

- The hardware recognizes two formats:

- Unsigned
  - All numbers are positive, a 1 in the most significant bit just means it is a really large number
  - Example: the unsigned int declaration in C/C++

- Signed
  - Numbers can be +/- , a 1 in the MSB means the number is negative
  - Example: the signed int or int declaration in C/C++

- Why would I need both?
  - To represent twice as many numbers when we're sure that we don't need negatives

Example: MIPS Instructions

- Example: consider a comparison instruction
  - `slt    $t0, $t1, $zero`
- and $t1 contains the 32-bit number
  - 11110111 11001010 00010100 00011110
- What gets stored in $t0?

Example: MIPS Instructions

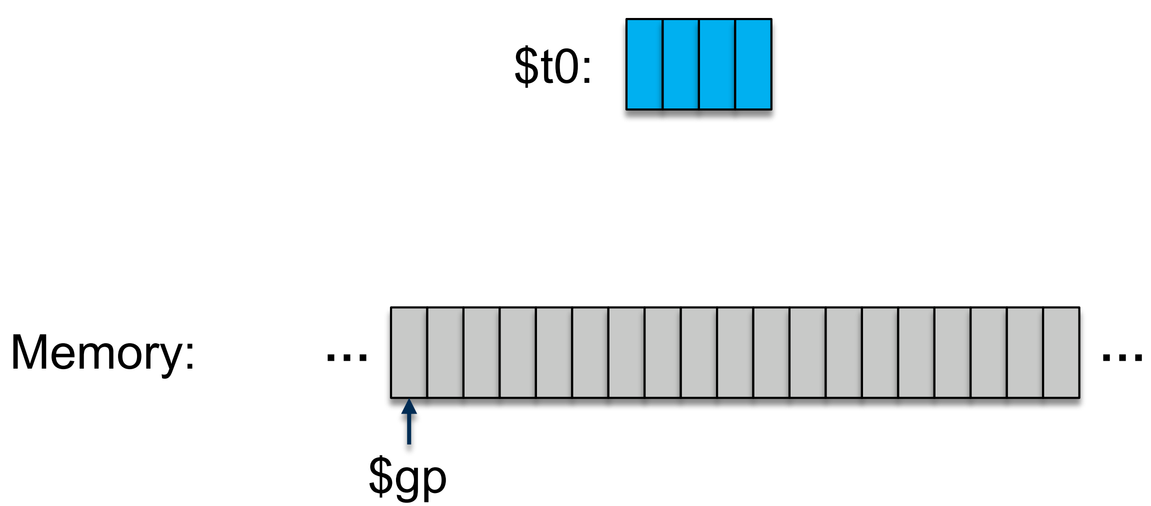
- Example: consider a comparison instruction
  - `slt    $t0, $t1, $zero`
- and $t1 contains the 32-bit number
  - 11110111 11001010 00010100 00011110
- What gets stored in $t0?

whether $t1 is a signed or unsigned number
the compiler/programmer must track this and accordingly
use either slt or sltu

```
slt   $t0, $t1, $zero   #stores  1 in $t0
sltu  $t0, $t1, $zero   #stores  0 in $t0
```

Recall: Dealing with Characters

- Instructions are also provided to deal with byte-sized and half-word quantities: lb (load-byte), sb, lh, sh
- Example: loading a byte from memory
  - Is the byte signed or unsigned?



Sign Extension

- Signed 8-/16-bit numbers must be converted into 32-bit signed numbers
  - Example:
    - `addi  $s0, $zero, 0x8000`
    - `addi  $s0, $zero, 0x4000`
- Conversion: take the most significant bit and use it to fill up the additional bits on the left

```
11111111 11111111 10000000 00000000 = -32768
00000000 00000000 01000000 00000000 = 16384
```

Unsigned Conversion

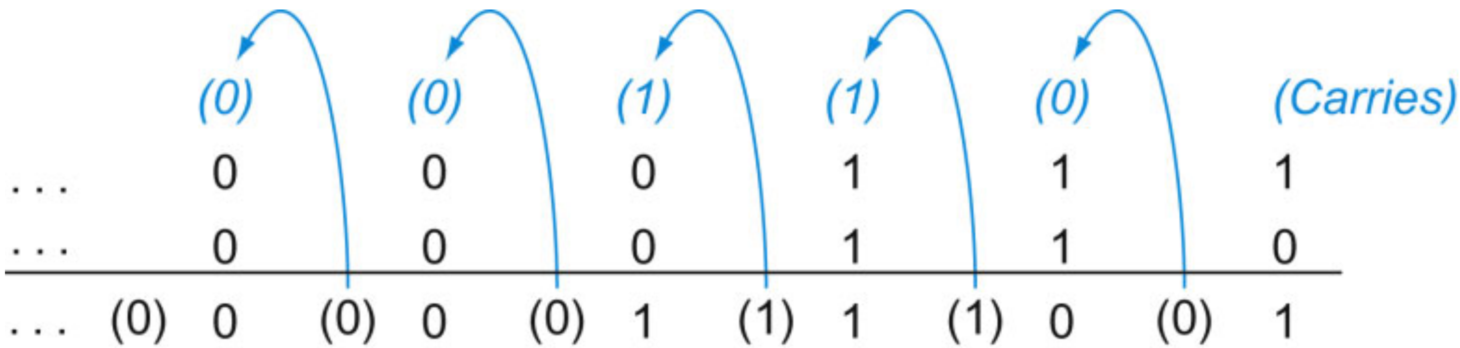
- Unsigned 8-/16-bit numbers must be converted into 32-bit signed numbers
  - Example:
    - `addiu $s0, $zero, 0x8000`
    - `addiu $s0, $zero, 0x4000`
- Conversion: fill up the additional bits on the left with zeroes

```
00000000 00000000 10000000 00000000 = 32768

00000000 00000000 01000000 00000000 = 16384
```

Addition and Subtraction

- Addition is similar to decimal arithmetic



- For subtraction, simply add the negative number
  - 4-bit example: 6 – 5 = 6 + (-5)

```
     0 1 1 0
+    1 0 1 1
```

Overflows

- Note: machines have limited numbers of bits for representing each number

- For an unsigned number, overflow happens when the last carry (1) cannot be accommodated

- For a signed number, overflow happens when the most significant bit is not the same as every bit to its left
  - when the sum of two positive numbers is a negative result
  - when the sum of two negative numbers is a positive result
  - The sum of a positive and negative number will never overflow